Written Exam for the B.Sc. or M.Sc. in Economics – Summer School 2013

Corporate Finance and Incentives

Final Exam/ Elective Course/ Master's Course

31st July 2013

(3-hour closed book exam)

Please note that the language used in your exam paper must correspond to the language of the title for which you registered during exam registration. I.e. if you registered for the English title of the course, you must write your exam paper in English. Likewise, if you registered for the Danish title of the course or if you registered for the English title, which was followed by "eksamen på dansk" in brackets, you must write your exam paper in Danish.

If you are in doubt about which title you registered for, please see the print of your exam registration from the students' self-service system.

The exam consists of 5 problems. All problems must be solved. The approximate weight in the final grade of each problem is stated. A problem can consist of different sub questions that do not necessarily have equal weight. Please provide intermediate calculations. Please give intuitive explanations and comment on your results. Good luck ☺

Problem 1 (Various themes, 15%)

1) How is the tangent portfolio found and why must this be the market portfolio?

The tangent portfolio is found by setting up a system of equations – one equation for each security – and solving for the portfolio weights when the risk premium ($r_i - r_f$) for each security is equal the covariance between the security and the tangency portfolio. The tangency portfolio is then the vector of weights scaled down by a factor so that the sum of weights equal to one.

If the tangency portfolio is not the market portfolio investors can do better by trading closer to the tangency portfolio, which would be an arbitrage. So a no arbitrage argument establishes the tangency portfolio as the market portfolio.

2) "Because all stocks are located on the Capital Market Line (CML) in CAPM they all have the same Sharpe ratio". Is this true or false? Explain.

False! All stocks are not located on the CML but on the Securities Market Line (SML). So all stocks do NOT have the same Sharpe ratio.

3) What is the value of the real option to wait one round to make the decision whether to invest or not, if the world can take two states one time period later?
   - Initial investment €100 million
   - Perpetual cash flow in good state: €40 million
   - Perpetual cash flow in bad state: €0
   - Probability of good state: 90%
   - Discount rate: 20%
   - Risk neutral investor

The value of the project without a real option is: $NPV_{No\ real\ option} = -100 + 0.9 \cdot 40 / 20\% = 80$

The value of the project with a real option: $NPV_{Real\ option} = 0.9 \cdot \left(-100 + 40 / 20\%\right) \cdot \left(1.2\right)^{-1} = 75$

Which means that the real option has no value because the cost of waiting (discounting) exceeds the value of additional information by waiting.

Problem 2 (Multi factor models and APT, 20%)

Assume that the following two-factor model can describe the return of the three stocks a, b and c:

$$r_a = .10 + F_1 + F_2 + \varepsilon_a$$
$$r_b = .11 + 2F_1 + F_2 + \varepsilon_b$$
$$r_c = .12 + F_1 + 2F_2 + \varepsilon_c$$

With all the usual assumptions.

1) Find the pure factor portfolios.
2) If the risk free rate is 7%, what are then the pure factor equations and what are the risk premiums λ_1 and λ_2?

We solve the three equations with regard to both $(\beta_1, \beta_2) = (1,0) \wedge (\beta_1, \beta_2) = (0,1)$

$$x_a + 2x_b + x_c = 1/0$$
$$x_a + x_b + 2x_c = 0/1$$
$$x_a + x_b + x_c = 1$$
$$\Rightarrow (x_a, x_b, x_c) = (2,0,-1)/(2,-1,0)$$

This gives us the following expected returns and pure factor equations:

$$\alpha_P = x_a \cdot \alpha_a + x_b \cdot \alpha_b + x_c \cdot \alpha_c$$
$$\alpha_{P1} = 2 \cdot 0.10 - 1 \cdot 0.12 = 0.08 \Rightarrow \lambda_1 = \alpha_{P1} - r_f = 0.01$$
$$\alpha_{P2} = 2 \cdot 0.10 - 1 \cdot 0.11 = 0.09 \Rightarrow \lambda_2 = \alpha_{P2} - r_f = 0.02$$

$$R_{P1} = 0.08 + F_1$$
$$R_{P2} = 0.09 + F_2$$

3) A new stock also follows the two-factor model: $r_x = \alpha_x + 2F_1 + 2F_2 + \varepsilon_x$. What is the value of α_x if there is no arbitrage?

We make a tracking portfolio consisting of two pure factor portfolio 1 and two pure factor portfolio 2 since these are the betas in the factor model. Our total portfolio weights must sum to one, which means that we must go three times short in the risk free asset. This gives us the expected return of the new stock and thus the alfa of that stock.

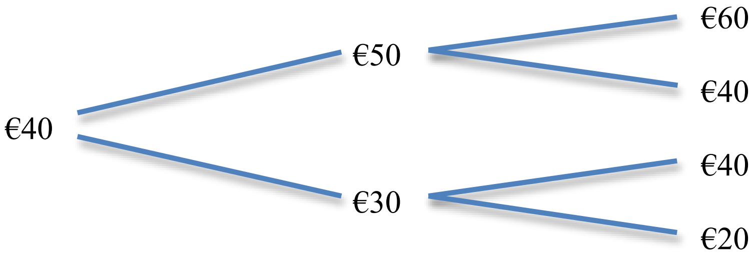
$$\alpha_x = (1 - x_{P1} - x_{P2}) \cdot r_f + x_{P1} \cdot \alpha_{P1} + x_{P2} \cdot \alpha_{P2} = -3 \cdot 0.07 + 2 \cdot 0.08 + 2 \cdot 0.09 = 0.13$$

4) What are usually the expected values of F_1 and F_2? Explain why.

Usually the expected value of the factors are zero, which means – in the case of macro factors – that it would be e.g. unexpected inflation changes, unexpected unemployment changes, unexpected interest rate changes etc. This is done to make interpretation easier and in general make the model easier to deal with

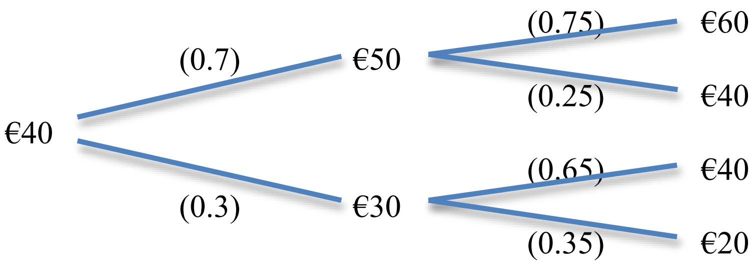
Problem 3 (Options, 20%)

A non-dividend paying stock currently cost €40. In the next two periods it can either increase in price by €10 or decrease in price by €10 each period as shown in the binary tree below. The risk free rate is 10% per period. An American call option on this stock has an exercise price of €35.

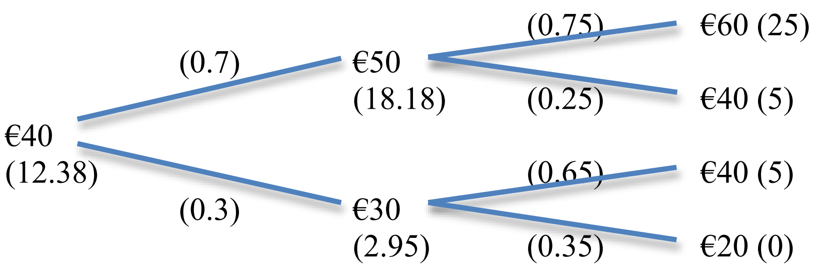


1) Assign risk free probabilities to the binary tree.

The risk free probabilities are found as: $\rho = \dfrac{(1 - r_f)S - S_d}{S_u - S_d}$, values inserted in the diagram below.



2) Use the risk free probabilities to price the call option.



Values of options are in brackets by the stock prices. E.g. the value of the option at a stock price of €50 is: C = ((0.75*25)+(0.25*5))/1.1 = 18.18

3) Use replication portfolios to price the call option.

$\Delta = \dfrac{C_u - C_d}{S_u - S_d}$ and $B = \dfrac{C_d - S_d\Delta}{1 + r_f}$, which gives the value of the option as: $C = S\Delta + B$

$$S = 50:\ \Delta = \frac{25-5}{60-40} = 1,\ B = \frac{5-40}{1.1} = -31.82 \Rightarrow C = 50 - 31.82 = 18.18$$

$$S = 30:\ \Delta = \frac{5-0}{40-20} = 0.25,\ B = \frac{0-20\cdot 0.25}{1.1} = -4.55 \Rightarrow C = 30\cdot 0.25 - 4.55 = 2.95$$

And finally we can price the option at the starting node:

$$S = 40:\ \Delta = \frac{18.18-2.95}{50-30} = 0.76,\ B = \frac{2.95-30\cdot 0.76}{1.1} = -18.08 \Rightarrow C = 40\cdot 0.76 - 18.08 = 12.38$$

4) Explain why an American put option normally can be hard to price using the put-call parity and then explain whether you think it can be used here in this particular setup. Explain your assumptions.

The put-call parity for non-dividend paying stocks gives the price of a European put as:

$$P = \underbrace{K - S}_{\text{Intrinsic value}} \underbrace{-dis(K) + C}_{\text{Time value}}.$$

The reason this *might not* work for an American put option is that the time value can get negative in which case it will be optimal to exercise prematurely.

In this particular case the only time it could be potentially optimal to exercise prematurely is when the stock price is €30, in which case the payoff at time 1 would be the intrinsic value at that time equal to 35-30=5. The value of the European put option at the same time is:

$$P = C + PV(K) - S = 2.95 + 35\cdot(1.1)^{-1} - 30 = 4.77$$

This indicates that it will be a good idea to exercise prematurely because the value from this is 5 where the value of the option is only 4.77. This is because the time value is negative and can be found by as: $Time\ value = PV(K) - K + C = 35\cdot(1.1)^{-1} - 35 + 2.95 = -0.23$

Conclusion: The put-call parity does not work here because we do in fact wish to exercise prematurely, which is not “an option” with European options.

Problem 4 (Fixed Income, 20%)

Explain what Fixed Income theory is. In your explanation you should include:

1. Bonds and valuation
2. Complete markets
3. Zero coupon bonds
4. Synthetically creating bonds and bootstrapping
5. Yield to maturity
6. Term structure
7. Spot rates, forward rates, discount factors
8. Types of bonds
9. Duration
10. Convexity

Problem 5 (Capital structure, 25%) – See also appendix 1

A firm has the following perpetual cash flow:

Turnover	1000
Operating costs	300
EBITDA	700
Depreciation & Amortization	300
EBIT	400
Interest payments	100
Earnings before taxes (EBT)	300
Tax (25%)	75
Profit after tax	225

The debt rate is 5% and return on equity at the current debt level is 15%

1) Find the unlevered asset return and the value of the unlevered firm.

The value of equity is found as E = 225 / 15% = 1,500. D = 100 / 5% = 2,000 and the value of the tax shield is 500 (tax rate * debt). This gives a value of the unlevered firm of 3,000.

The unlevered asset return is:

$$r_A = \left(\frac{E}{E+D(1-T_C)}\right)r_E + \left(\frac{D(1-T_C)}{E+D(1-T_C)}\right)r_D = \left(\frac{1,500}{3,000}\right)\cdot 15\% + \left(\frac{1,500}{3,000}\right)\cdot 5\% = 10\%$$

2) What is WACC at the current debt level? Explain why WACC changes when leverage is added and relate it to Modigliani-Miller.

$$r_L = \left(\frac{E}{E+D}\right)r_E + \left(\frac{D}{E+D}\right)(1-\tau)r_D = \left(\frac{1,500}{3,500}\right)\cdot 15\% + \left(\frac{2,000}{3,500}\right)\cdot(1-25\%)\cdot 5\% = 8.57\%$$

The WACC decreases with leverage because the tax shield makes the cost of capital cheaper. Modigliani-Miller assume perfect capital markets. Here we simply correct for the effect of taxes.

Now assume that the cash flow is a growing perpetuity with a constant growth rate of 2.5% and debt-to-equity is kept constant over time

3) What is the new value of the tax shield, equity, debt and the firm?

Value of tax shield: $V_{Tax} = \frac{\tau D r_D}{r_D - g} = \frac{25\% \cdot 2{,}000 \cdot 5\%}{5\% - 2.5\%} = 1{,}000$

Value of unlevered firm: $V_U = \frac{EBIT\left(1-\tau\right)}{r_U - g} \frac{300}{10\% - 2.5\%} = 4{,}000$

Since the value of the levered firm is the value of the unlevered firm plus the value of the tax shield the new value of the levered firm is 5,000. With debt of 2,000 the residual equity must be 3,000

Appendix 1: Un-levering and de-levering (refer to problem 5)

The (Asset) return of the unlevered firm (de-levering / un-levering):

$$r_A = \left(\frac{E}{E + D(1-T_C)}\right) r_E + \left(\frac{D(1-T_C)}{E + D(1-T_C)}\right) r_D$$

The (Asset) beta of the unlevered firm (de-levering / un-levering):

$$\beta_A = \left(\frac{E}{E + D(1-T_C)}\right) \beta_E + \left(\frac{D(1-T_C)}{E + D(1-T_C)}\right) \beta_D$$

Re-levering:

$$\beta_E = \left(\frac{E + D(1-T_C)}{E}\right) \beta_A - \left(\frac{D(1-T_C)}{E}\right) \beta_D = \beta_A + \left(\frac{D(1-T_C)}{E}\right)(\beta_A - \beta_D)$$

$$r_E = \left(\frac{E + D(1-T_C)}{E}\right) r_A - \left(\frac{D(1-T_C)}{E}\right) r_D = r_A + \left(\frac{D(1-T_C)}{E}\right)(r_A - r_D)$$

Where subscript "A" refer to (unlevered) Assets, "E" Equity, "D" Debt, and "T_C" is the corporate tax rate